

Proper placement of the minor seventh in untempered (just) intonation

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Once the domain of keyboardists, the subject of temperament and tuning is becoming more and more of a concern to wind musicians, particularly trombonists. This is evident in the release of publications specifically for trombones that address this issue, such as David Schwartz's *Breakfast* (<http://mysite.verizon.net/~dschwar/breakfast.html>), and my own *Intonation Studies* (<http://www.tenorposaune.com/intonation.php>). These resources are, of course, in addition to Stephen Colley's seminal *TuneUp* (<http://www.tuneupsystems.com/>), which is available for any instrument. While it is nice to see intonation being treated seriously in the educational literature, students without a solid technical background might be confused by the variety of information available. The reason for this is plain: all three of the books I listed disagree with each other.

There is a consensus on how to tune a diatonic major scale, but some chromatic intervals are more problematic; the minor seventh is an interval that is the subject of constant debate. *TuneUp* lists the interval at a frequency ratio of $7/4$, or about 31 cents lower than its equal-tempered equivalent. *Breakfast* lists the interval a quarter step higher, at 18 cents higher than the equal tempered interval (frequency ratio $9/5$). *Intonation Studies* is in the middle, with a frequency ratio of $16/9$, 4 cents lower than an equal tempered minor 7th. While it would be convenient to write an article on why my book is

correct and the others aren't, the issue is more complicated than that. All three frequency ratios have appropriate applications.

If a minor *ii* chord is built using the commonly accepted interval ratios for a diatonic major scale ($9/8$, $4/3$, $5/3$), one immediately notices that unlike every other diatonic chord, the internal intervals are not justly tuned. The minor third is about six cents lower than an equal tempered minor third, rather than the expected sixteen cents higher. The outside fifth is almost twenty cents lower than an equal tempered fifth, rather than two cents higher. This chord is very dissonant. And yet, if a minor seventh is added to this triad (scale degree 8, ratio $2/1$), the major triad on the top of the chord is tuned perfectly. The note at fault here is the root, the second scale degree. It is tuned at $9/8$ because this makes it fit in well as the fifth of a *V* chord, but this isn't necessarily the proper tuning in all circumstances. If the pitch is lowered to a frequency ratio of $10/9$ (18 cents lower than an equal tempered major second), then all the intervals of the *ii* chord are justly tuned as they should be ($6/5$, $3/2$). The minor seventh then has a frequency ratio of $9/5$ with the root of the chord, and is about 18 cents higher than the equal tempered minor seventh.

In brief, the listed frequency ratio of $9/8$ for the major second is primarily intended for dominant functioning applications. In many subdominant situations, a frequency ratio of $10/9$ is more appropriate; this gives rise to the $9/5$ frequency ratio for the minor seventh. In subdominant applications, therefore, a high minor seventh is appropriate.

TuneUp's listed $7/4$ frequency ratio has the smallest integers and is therefore the most consonant version of this interval. It coincides with the 7th partial in the overtone series (the Ab in first position on the trombone), and is very flat compared to equal temperament. It is

so incredibly low that I can find no use for it as a minor seventh. One must remember, however, that enharmonicism does not exist in untempered intonation. Ten semitones in equal temperament can be a minor seventh, but it can also be an augmented sixth. The $7/4$ frequency ratio belongs in this situation. In an augmented sixth, the top note (raised fourth scale degree) acts as the leading tone to the fifth scale degree. Leading tones in just intonation are always lower than their equal tempered equivalents, so having a particularly low pitch makes sense in this context. Also, the bass of an augmented sixth chord is the flat sixth scale degree, which is already relatively high. If these notes are all tuned according to a normal minor sixth and augmented fourth, the interval is $225/128$, or about 23 cents (a quarter of a semitone) lower than the equal tempered interval. However, the integers in that ratio are very large; it is a highly dissonant chord. Tuning the interval to $7/4$ preserves the narrowness of the interval and the tension that this narrowness causes in the voice leading, but removes the dissonance.

Knowing that these tunings have their place, I still opted to use $16/9$ in my *Intonation Studies*. This interval is the most dissonant of the three and falls roughly near the equal tempered version. I listed this interval rather than the others because it is best used in a dominant context, and that is the most common usage of the minor seventh. The $16/9$ ratio results from the interval between the untempered fifth and fourth scale degrees: the root and the seventh in a V^7 chord. Even though it is more dissonant than other tunings of the interval, it is still relatively consonant. It is a purer interval than most of the other chromatic pitches and is approximately as consonant as a justly tuned major seventh. In addition to the vertical intonation between simultaneously sounding pitches, people hear intonation

across a musical line, and these horizontal relationships are important in this interval.

Tuning a dominant chord with a $16/9$ seventh preserves the relationships between Sol-Do and Fa-Mi. Since these resolutions are at the heart of western tonal harmony, it is vital that they be as pure as possible.

Stephen Colley's *TuneUp*, David Schwartz's *Breakfast*, and my *Intonation Studies* all have advantages and disadvantages. They also all list a different optimal size for the minor seventh interval. Each of these sizes is appropriate in a different context: a small (-31c) interval is best used as an augmented sixth, while a wide (+18c) interval is good in subdominant chords. A moderate (-4c) interval works best in dominant chords. When using these books or any other music, performers should make educated decisions about how to tune this problematic interval based on how it is used. Unlike very consonant intervals like the perfect fifth, there is no one correct place to play a minor seventh; it is highly dependent on context.